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Tensor numerical methods for high-dimensional problems

Outline and References

- 1 High-dimensional approximation
- 2 Tensors and low-rank tensor formats
- 3 Tensor structure of high-dimensional equations
- 4 Low-rank methods for tensor-structured problems

• Approximation theory



Ronald A DeVore and George G Lorentz.
Constructive approximation, volume 303.
Springer Science & Business Media, 1993.



R. A. DeVore.
Nonlinear approximation.
Acta Numerica, 7:51–150, 1998.



V. Temlyakov.
Greedy approximation.
Acta Numerica, 17:235–409, 2008.

• Tensors



T. G. Kolda and B. W. Bader.
Tensor decompositions and applications.
SIAM Review, 51(3):455–500, September 2009.



W. Hackbusch.
Tensor spaces and numerical tensor calculus, volume 42 of *Springer series in computational mathematics*.
Springer, Heidelberg, 2012.



A. Defant and K. Floret.
Tensor norms and operator ideals.
North-Holland, Amsterdam New York, 1993.



R. A. Ryan.

Introduction to tensor products of Banach spaces.
Springer, 2002.

• Sparse tensors



H-J. Bungartz and M. Griebel.

Sparse grids.
Acta Numerica, 13:147–269, 2004.



C. Schwab and C. Gittelson.

Sparse tensor discretizations of high-dimensional parametric and stochastic pdes.
Acta Numerica, 20:291–467, 2011.



A. Chkifa, A. Cohen, and C. Schwab.

High-dimensional adaptive sparse polynomial interpolation and applications to parametric pdes.
Foundations of Computational Mathematics, 14(4):601–633, 2014.



Albert Cohen and Ronald DeVore.

Approximation of high-dimensional parametric pdes.
Acta Numerica, 24:1–159, 5 2015.

• Low-rank methods for tensor-structured equations



B. B. Khoromskij and C. Schwab.

Tensor-structured galerkin approximation of parametric and stochastic elliptic pdes.
SIAM Journal on Scientific Computing, 33(1):364–385, 2011.



B. Khoromskij.

Tensors-structured numerical methods in scientific computing: Survey on recent advances. *Chemometrics and Intelligent Laboratory Systems*, 110(1):1 – 19, 2012.



A. Nouy.

Low-rank methods for high-dimensional approximation and model order reduction. *ArXiv e-prints*, November 2015.



Markus Bachmayr, Reinhold Schneider, and André Uschmajew.

Tensor networks and hierarchical tensors for the solution of high-dimensional partial differential equations. *Foundations of Computational Mathematics*, pages 1–50, 2016.



Anthony Nouy.

Low-Rank Tensor Methods for Model Order Reduction, pages 1–26. Springer International Publishing, Cham, 2016.

- **Low-rank methods for tensor-structured equations: greedy algorithms**



E. Cances, V. Ehrlacher, and T. Lelievre.

Convergence of a greedy algorithm for high-dimensional convex nonlinear problems. *Mathematical Models & Methods In Applied Sciences*, 21(12):2433–2467, December 2011.



A. Falcó and A. Nouy.

Proper generalized decomposition for nonlinear convex problems in tensor banach spaces. *Numerische Mathematik*, 121:503–530, 2012.

- **Low-rank methods for tensor-structured equations: truncated iterations**



D. Kressner and C. Tobler.

Low-rank tensor Krylov subspace methods for parametrized linear systems.
SIAM Journal on Matrix Analysis and Applications, 32(4):1288–1316, 2011.



J. Ballani and L. Grasedyck.

A projection method to solve linear systems in tensor format.
Numerical Linear Algebra with Applications, 20(1):27–43, 2013.



M. Bachmayr and W. Dahmen.

Adaptive near-optimal rank tensor approximation for high-dimensional operator equations.
Foundations of Computational Mathematics, 15(4):839–898, 2015.



Markus Bachmayr and Reinhold Schneider.

Iterative methods based on soft thresholding of hierarchical tensors.
Foundations of Computational Mathematics, pages 1–47, 2016.

- **Preconditioners in low-rank formats**



B. Khoromskij.

Tensor-structured preconditioners and approximate inverse of elliptic operators in \mathbb{R}^d .
Constructive Approximation, 30(3):599–620, 2009.



L. Giraldi, A. Nouy, and G. Legrain.

Low-rank approximate inverse for preconditioning tensor-structured linear systems.
SIAM Journal on Scientific Computing, 36(4):A1850–A1870, 2014.

- **Higher-order singular value decompositions and related tensor truncation schemes**



L. De Lathauwer, B. De Moor, and J. Vandewalle.
A multilinear singular value decomposition.
SIAM J. Matrix Anal. Appl., 21(4):1253–1278, 2000.



Ivan V Oseledets and Eugene E Tyrtshnikov.
Breaking the curse of dimensionality, or how to use svd in many dimensions.
SIAM Journal on Scientific Computing, 31(5):3744–3759, 2009.



L. Grasedyck.
Hierarchical singular value decomposition of tensors.
SIAM J. Matrix Anal. Appl., 31:2029–2054, 2010.

• Sampling methods for low-rank approximation



J. Ballani, L. Grasedyck, and M. Kluge.
Black box approximation of tensors in hierarchical tucker format.
Linear Algebra and its Applications, 438(2):639 – 657, 2013.
Tensors and Multilinear Algebra.



I. Oseledets and E. Tyrtshnikov.
TT-cross approximation for multidimensional arrays.
Linear Algebra And Its Applications, 432(1):70–88, JAN 1 2010.



H. Rauhut, R. Schneider, and Z. Stojanac.
Tensor completion in hierarchical tensor representations.
ArXiv e-prints, April 2014.



M. Chevreuil, R. Lebrun, A. Nouy, and P. Rai.
A least-squares method for sparse low rank approximation of multivariate functions.
SIAM/ASA Journal on Uncertainty Quantification, 3(1):897–921, 2015.

- **Geometry**



S. Holtz, T. Rohwedder, and R. Schneider.

On manifolds of tensors of fixed tt-rank.
Numerische Mathematik, 120(4):701–731, 2012.



A. Uschmajew and B. Vandereycken.

The geometry of algorithms using hierarchical tensors.
Linear Algebra and its Applications, 439(1):133–166, 2013.



A. Falco, W. Hackbusch, and A. Nouy.

Geometric Structures in Tensor Representations (Final Release).
ArXiv e-prints, May 2015.

- **Approximability of tensors using low-rank or sparse formats**



A. Chkifa, A. Cohen, and C. Schwab.

Breaking the curse of dimensionality in sparse polynomial approximation of parametric pdes.
Journal de Mathématiques Pures et Appliquées, 103(2):400–428, 2015.



R. Schneider and A. Uschmajew.

Approximation rates for the hierarchical tensor format in periodic sobolev spaces.
Journal of Complexity, 30(2):56 – 71, 2014.
Dagstuhl 2012.